

REAL NUMBERS

Comprehensive Study Notes

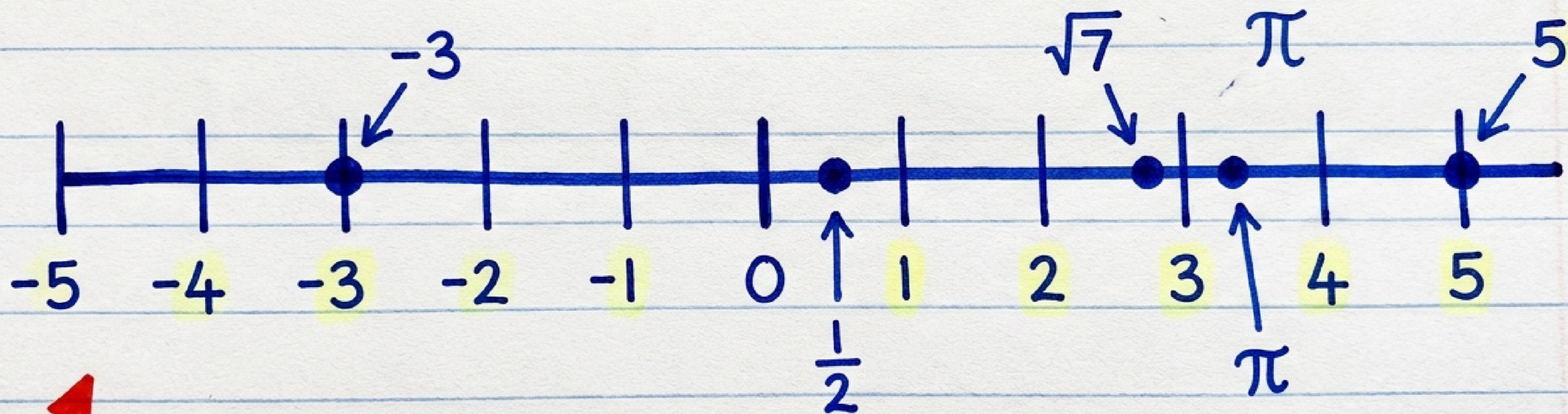
Class: Mathematics

Topic: Chapter 1

Study Guide • Definitions • Theorems • Examples

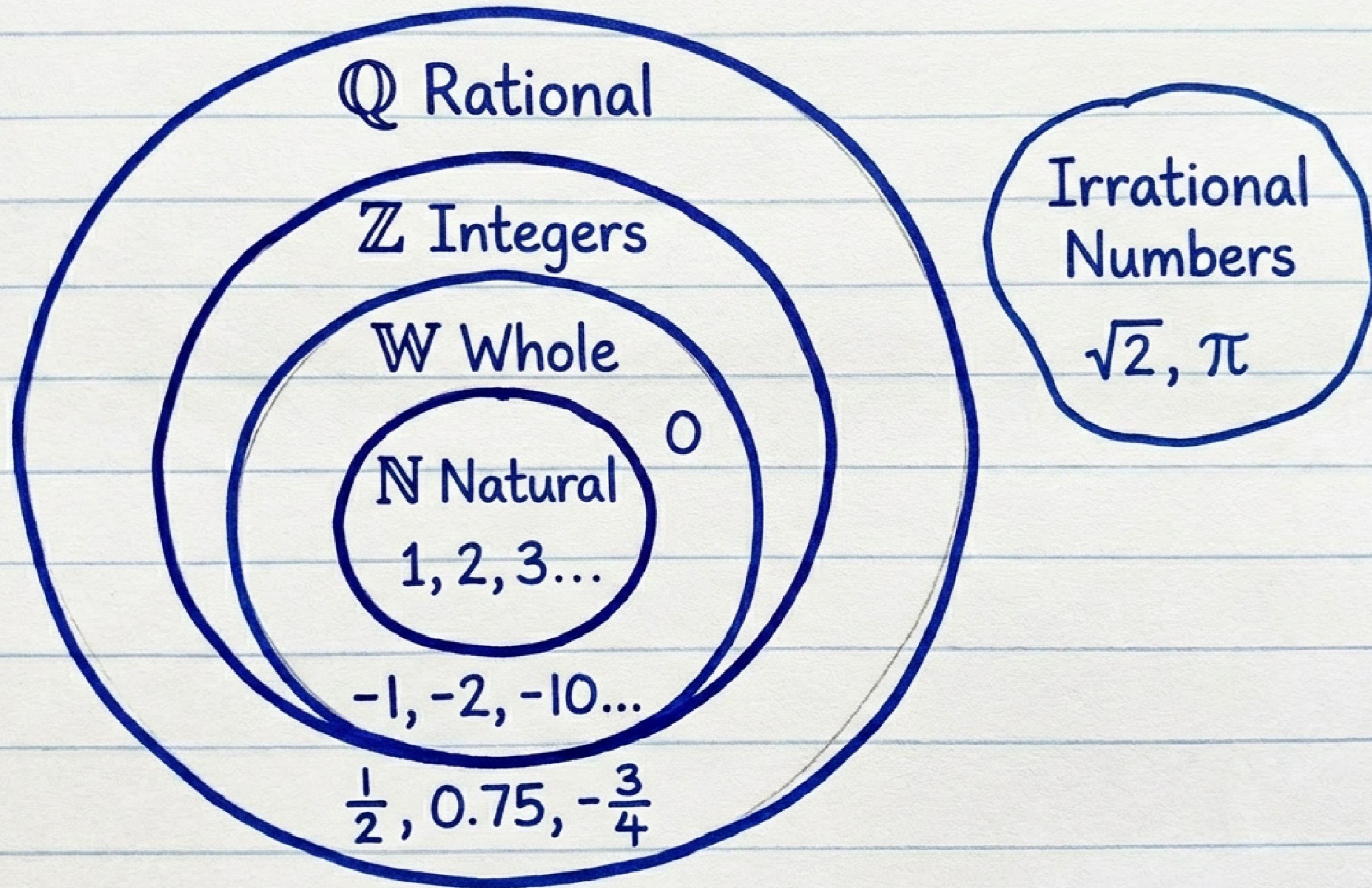
What are Real Numbers? (\mathbb{R})

Definition: All numbers that can be represented on a number line.



Each real number = A unique point on this line.

The Family Tree (Classification)



Rational numbers include
Integers, Wholes, and Integers,
Rational numbers include

Rational (\mathbb{Q}) vs. Irrational

Rational Numbers

- Can be written as $\frac{p}{q}$ ($q \neq 0$)
- Decimal: **Terminating** OR **Recurring**
- Examples: $\frac{3}{5}$, -2 , 0.25

Irrational Numbers

- **CANNOT** be written as $\frac{p}{q}$
- Decimal: **Non-terminating** AND **Non-repeating**
- Examples: $\sqrt{2}$, $\sqrt{3}$, π

Note: Rational numbers sit exactly on the number line; Irrationals are approximated.

Euclid's Division Lemma

For positive integers a and b , there exist unique integers q and r .

$$a = bq + r \quad 0 \leq r < b$$

Dividend

Divisor

Quotient

Remainder

Used
to
find
HCF

Translation: Dividend = (Divisor \times Quotient) + Remainder

Euclid's Division Algorithm (The Process)

Step 1: Apply Lemma ($a = bq + r$) to numbers.

Step 2: Check Remainder (r).

If $r = 0$

Divisor is
the HCF.
Stop.

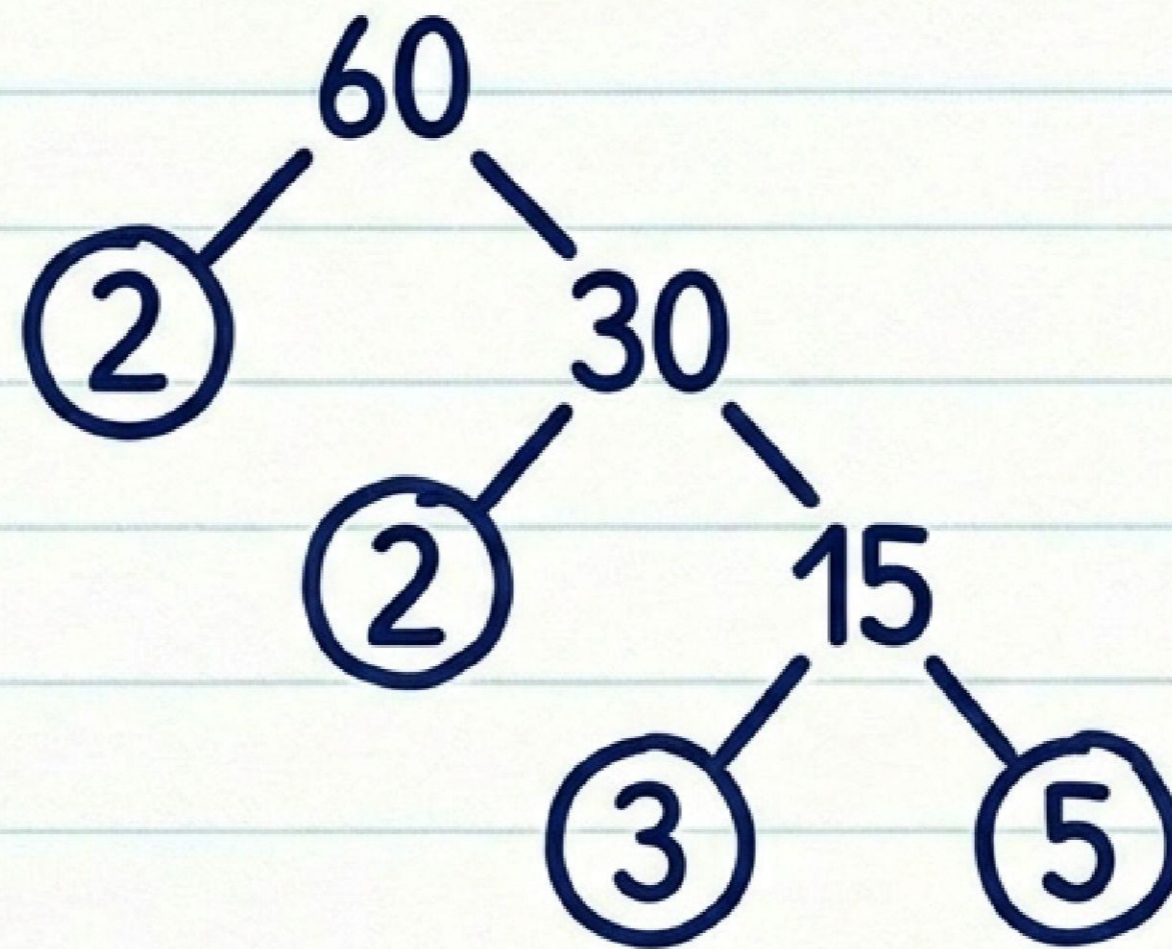
If $r \neq 0$

Repeat process. Old Divisor
becomes new Dividend.

Step 3: Continue until $r = 0$.

Fundamental Theorem of Arithmetic

Every composite number is a unique product of primes.



$$60 = 2 \times 2 \times 3 \times 5$$

$$60 = 2^2 \times 3 \times 5$$

Order doesn't matter, but factors are unique!

Method: Finding HCF & LCM

Example: 12 and 18

Step 1: Prime Factorisation

$$12 = 2^2 \times 3$$

$$18 = 2 \times 3^2$$

Step 2: HCF (Lowest Powers)

Take common primes with smallest exponent.

$$2^1 \times 3^1 = 6$$

Step 3: LCM (Highest Powers)

Take all primes with highest exponent.

$$2^2 \times 3^2 = 4 \times 9 = 36$$

Worked Example & The Golden Rule

☆ Example 2: 24 & 36

☆ $24 = 2^3 \times 3$

$$36 = 2^2 \times 3^2$$

$$\text{HCF} = 2^2 \times 3 = 12$$

$$\text{LCM} = 2^3 \times 3^2 = 8 \times 9 = 72$$

Product of Numbers =

$$24 \times 36 = 864$$

HCF \times LCM

$$= 12 \times 72$$

$$= 864$$

HCF \times LCM = Product of the numbers

☆ Warning: Only works for TWO numbers!

Decimal Expansion: Terminating?

☆ For rational number $\frac{p}{q}$ (in lowest form), look at the denominator q .



If prime factors of q are **ONLY 2s or 5s**
→ **TERMINATING.**

☆ Example 1: $\frac{1}{8}$

Denom: $8 = \underline{2^3}$ (Only 2s)

↳ 0.125 (Terminating)

☆ Example 2: $\frac{7}{40}$

Denom: $40 = \underline{2^3} \times 5$ (Only 2s and 5s) ☆

↳ 0.175 (Terminating) ☆

Decimal Expansion: Non-Terminating?

☆ If q has ANY prime factor other than 2 or 5.

Intruder factor present \rightarrow NON-TERMINATING
RECURRING.



☆ Example 1: $\frac{1}{3}$

Example 2: $\frac{5}{6}$

Example 3: $\frac{2}{7}$

Denom: 3

Denom: $6 = 2 \times \underline{3}$

Denom: 7

☆ \rightarrow 0.333... (Recurring)

\rightarrow 0.285714... (Recurring)



Properties of Real Numbers

Closure

Real numbers are closed under Addition (+) and Multiplication (×).

Commutative

$$a + b = b + a$$

$$a \times b = b \times a$$




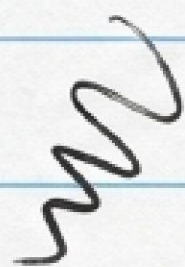
Associative

$$(a + b) + c = a + (b + c)$$

Distributive

$$a(b + c) = \underline{ab + ac}$$

Summary Checklist

- \mathbb{R} includes Rationals (p/q) & Irrationals (non-repeating).
- Euclid's Lemma: $a = bq + r$ (used for HCF). 
- Fundamental Theorem: Primes build all Composite numbers. 
- Formula: $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$. 
- Denominator with only 2s/5s = Terminating Decimal. 

Quick Quiz (Part 1)

Q1: Which is an Irrational number?

$\frac{2}{3}$, 0.25 , $\sqrt{5}$, -4

Q2: Smallest Whole Number?

0 , 1 , -1

Q3: Euclid Lemma for $a=25$, $b=4$?

$$25 = 4 \times 6 + 1 \quad \checkmark$$

Quick Quiz (Part 2)

Q4: Prime factorisation of 36?

Options: 2×18 , 6×6 , $2^2 \times 3^2$

Q5: Which fraction gives a terminating decimal?

Options: $1/3$, $1/6$, $1/8$, $1/7$

Reason: $8 = 2^3$ (Only powers of 2)

Good luck with your study! 😊