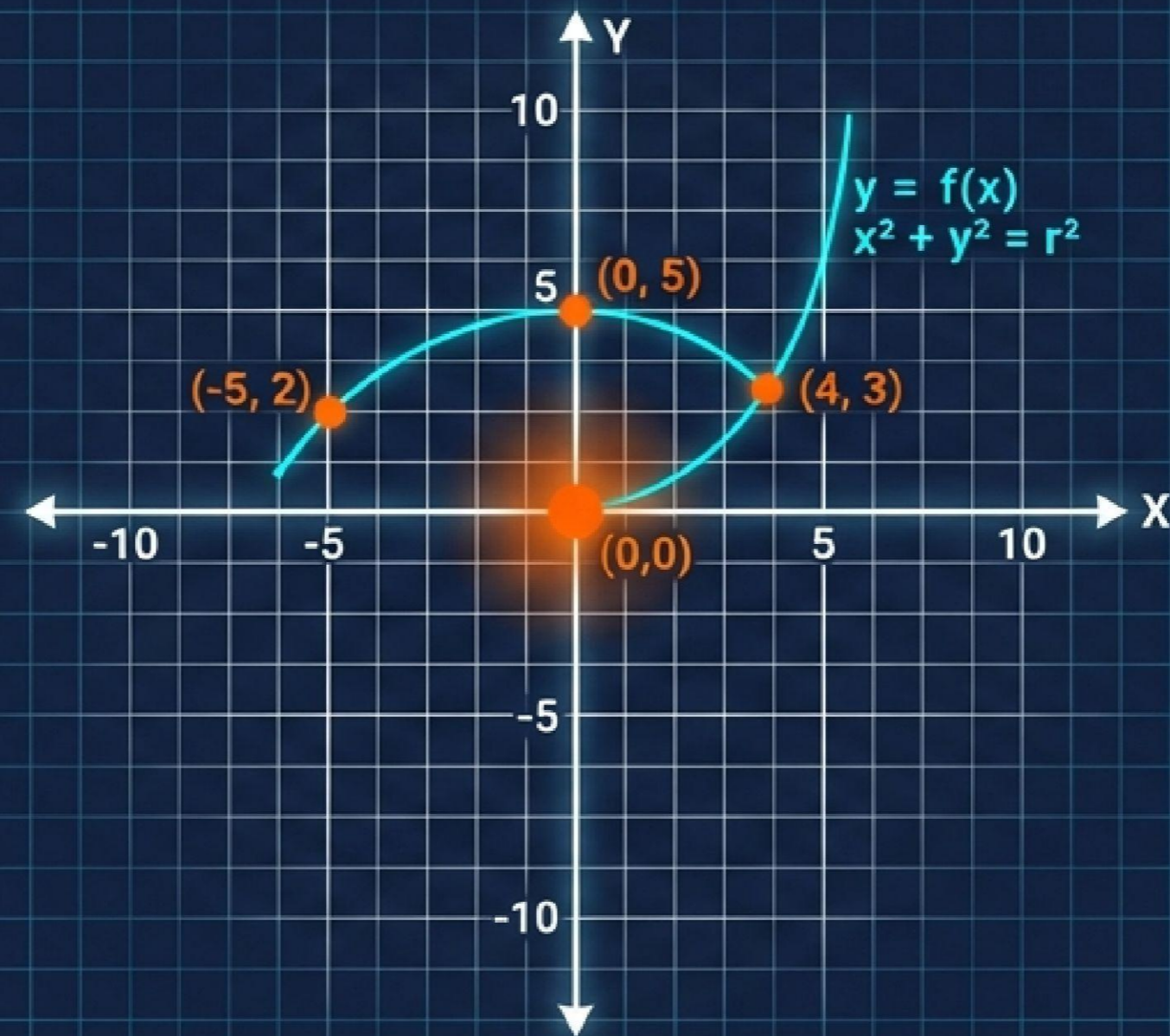


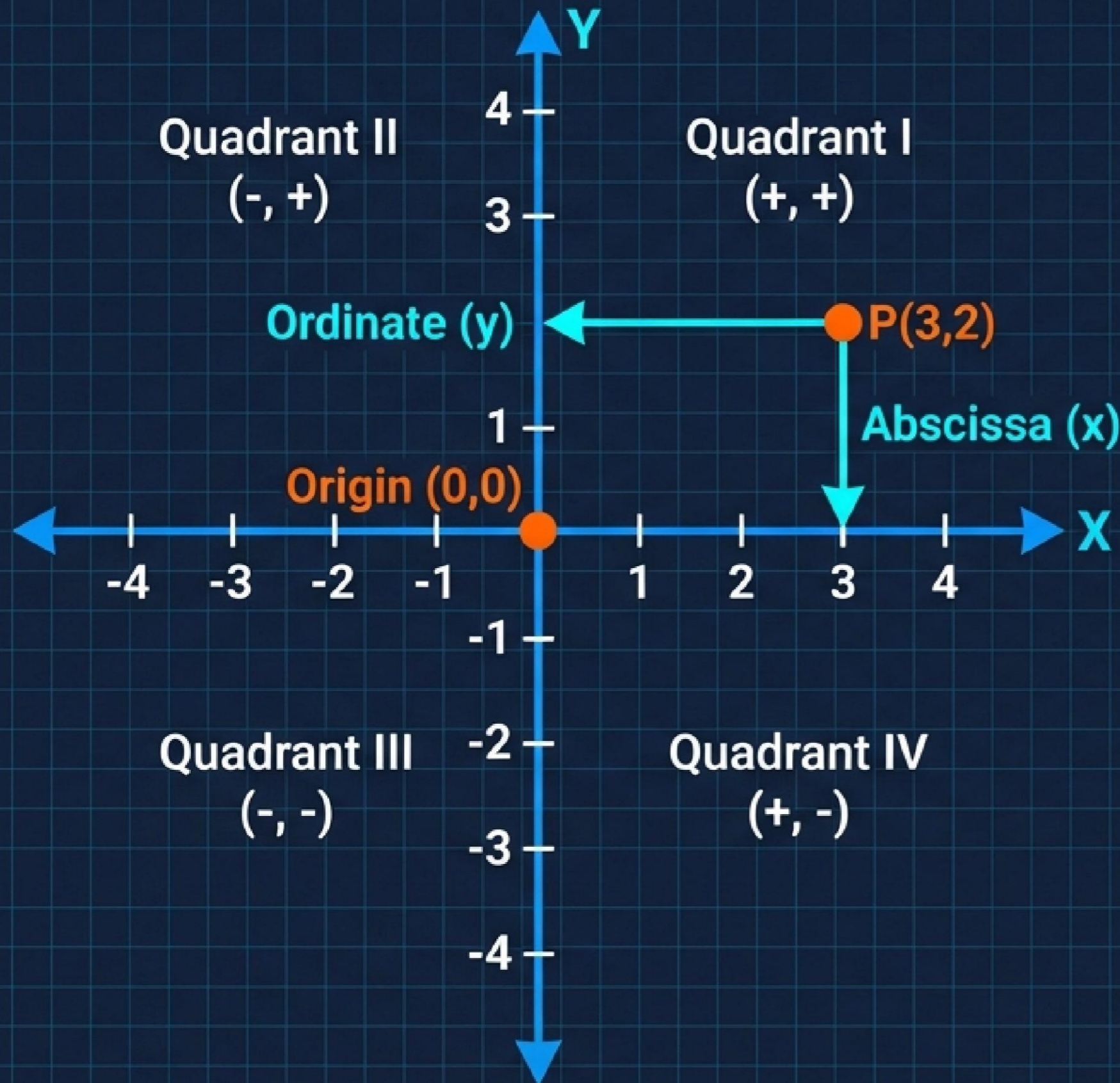
COORDINATE GEOMETRY: BRIDGING ALGEBRA AND SPACE

A dimensional journey through the Cartesian Plane.



Coordinate geometry is the system where the position of points on a plane is described using ordered pairs of numbers. It is the mathematical framework that translates geometric shapes into algebraic equations, forming the foundation for engineering, physics, and computer science.

THE CARTESIAN ARCHITECTURE



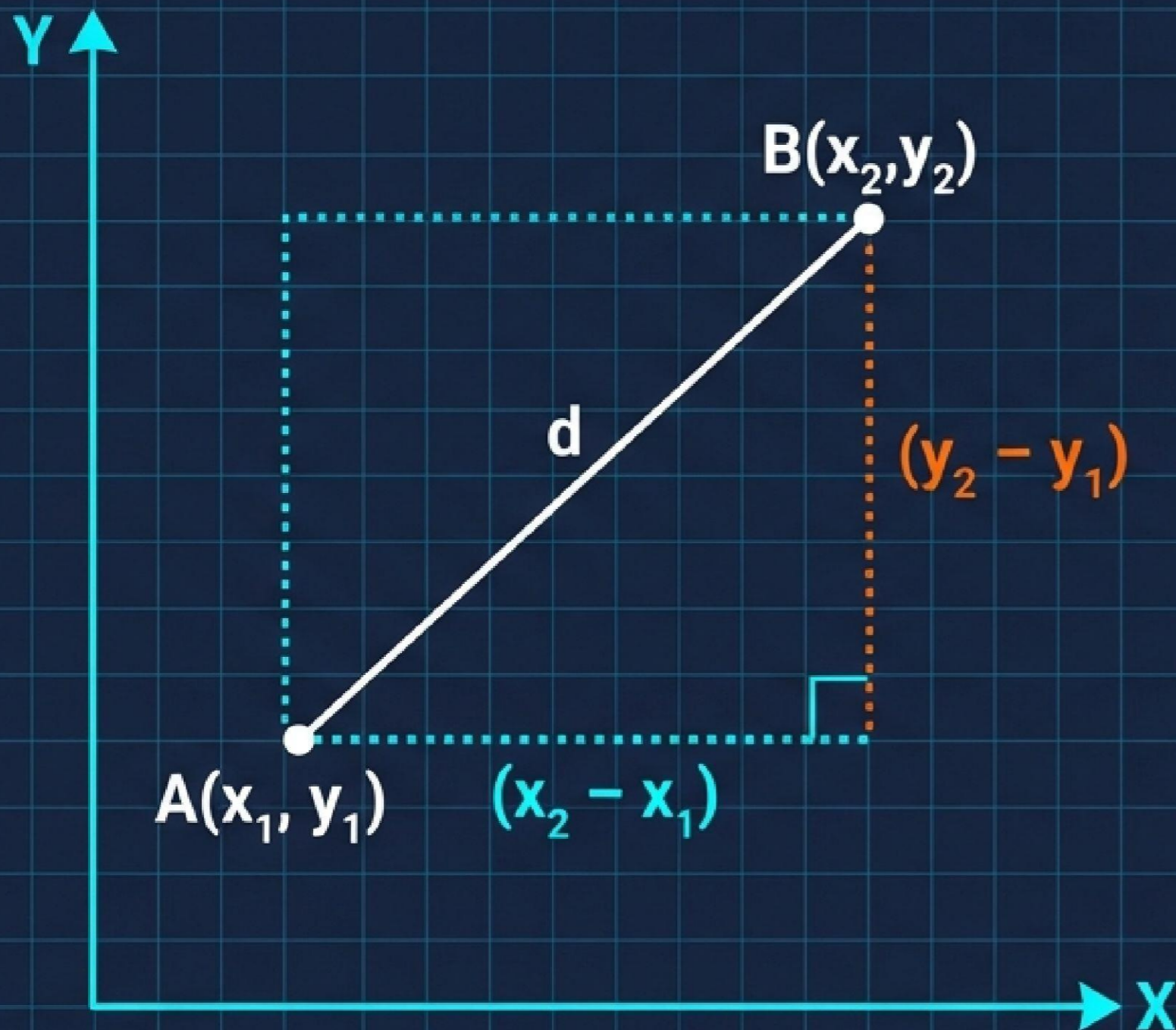
The Address: Every location is a unique ordered pair (x, y) .

Abscissa (x): Horizontal distance from the y-axis.

Ordinate (y): Vertical distance from the x-axis.

Origin: The intersection point $(0, 0)$, the anchor for all measurements.

MEASURING DISTANCE



Logic: Treat the line as the hypotenuse of a right triangle.

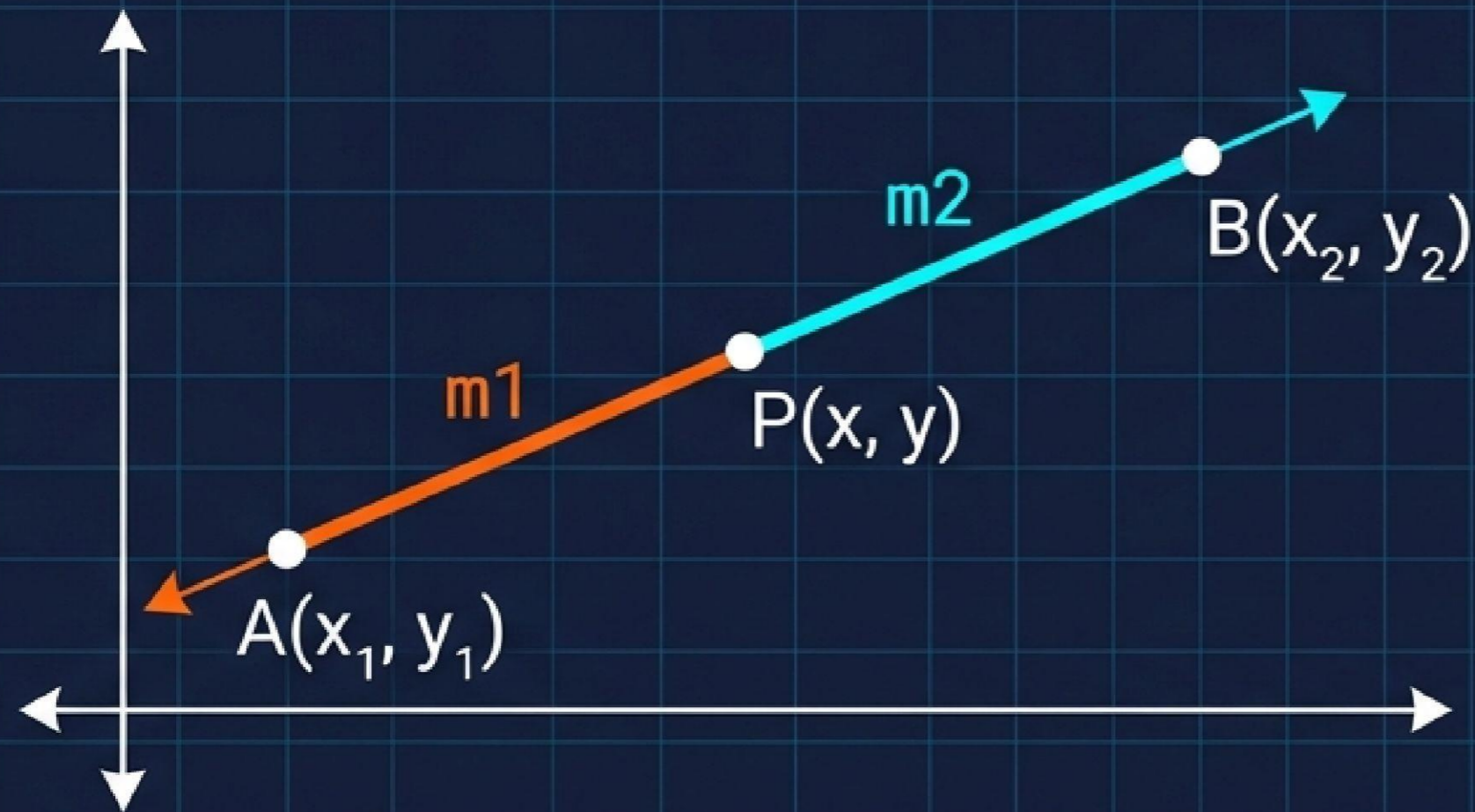
Horizontal leg: $(x_2 - x_1)$

Vertical leg: $(y_2 - y_1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Special Case: Distance from Origin $(0,0)$ simplifies to $\sqrt{x^2 + y^2}$.

THE SECTION FORMULA



$$P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

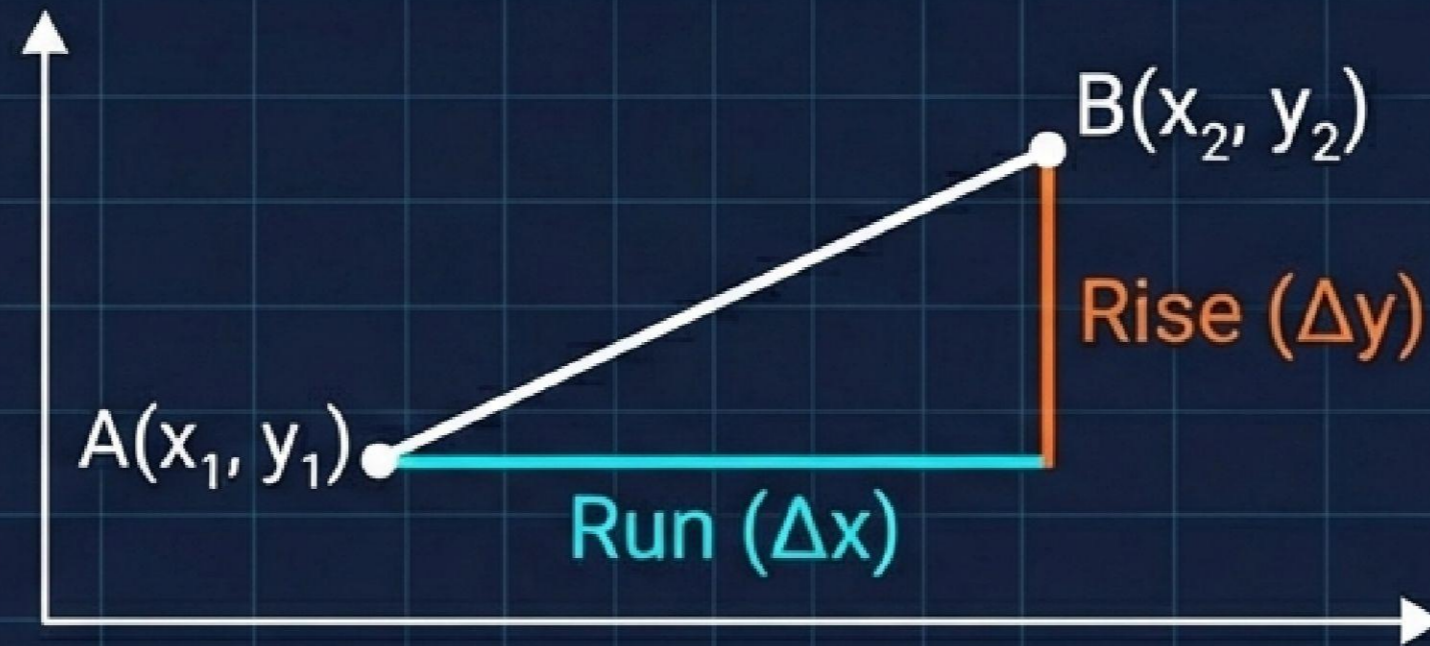
SPECIAL CASES

1. Midpoint Formula
(Ratio 1:1):

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

2. Problem Solving Tip:
If ratio is unknown,
assume k:1 to solve
for a single
variable.

DEFINING STEEPNESS: THE SLOPE



Slope (m) represents the change in y with respect to x .

1. Rise over Run: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

2. Trigonometric: $m = \tan \theta$



Positive (+)



Negative (-)



Zero Slope



Undefined

LINE RELATIONSHIPS

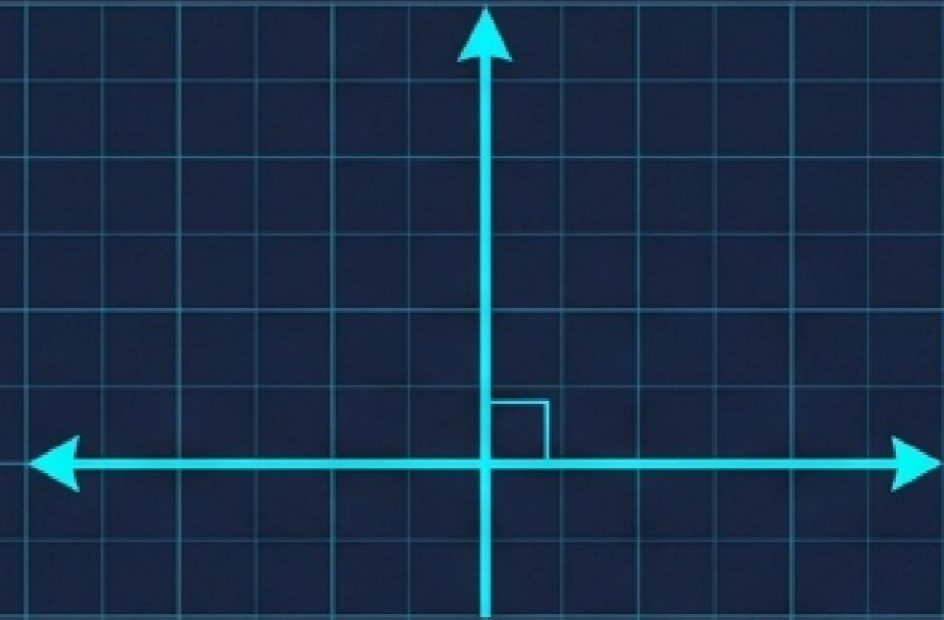
PARALLEL LINES



Condition: Inclinations are identical.

$$m_1 = m_2$$

PERPENDICULAR LINES

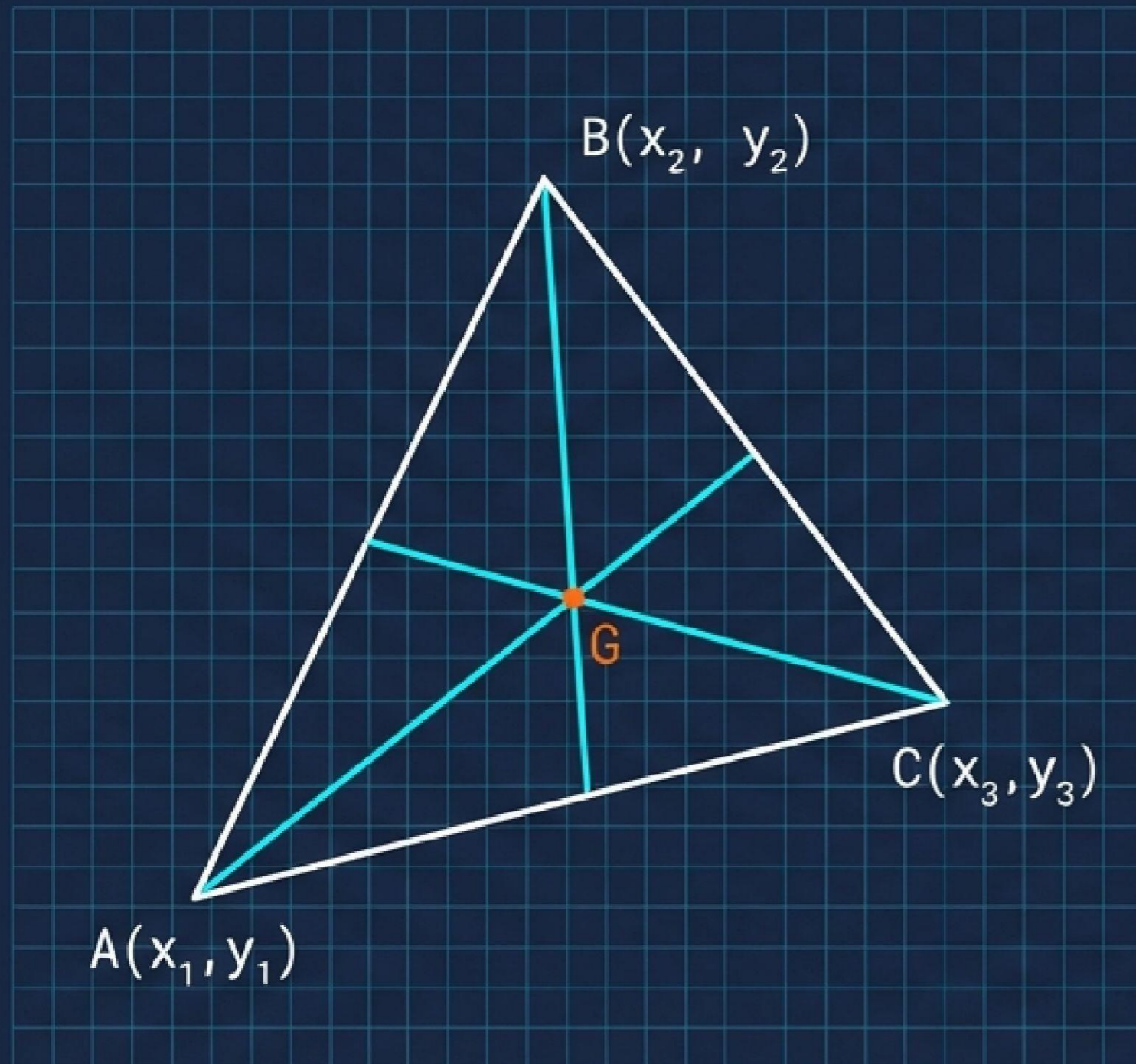


Condition: Slopes are negative reciprocals.

$$m_1 \times m_2 = -1$$

$$\text{Angle Between Lines formula: } \tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

THE TRIANGLE: AREA AND CENTROID



AREA

$$\text{Area} = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

*Absolute value ensures area is never negative.

CENTROID

Centroid Formula

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

THE COLLINEARITY CONDITION



Three points lie on the same straight line if:

Distance Method

$$\overline{AB} + \overline{BC} = \overline{AC}$$

(Sum of segments = Total length)

Slope Method

$$\text{Slope of } \overline{AB} = \text{Slope of } \overline{BC}$$

(Steepness is constant)

Area Method

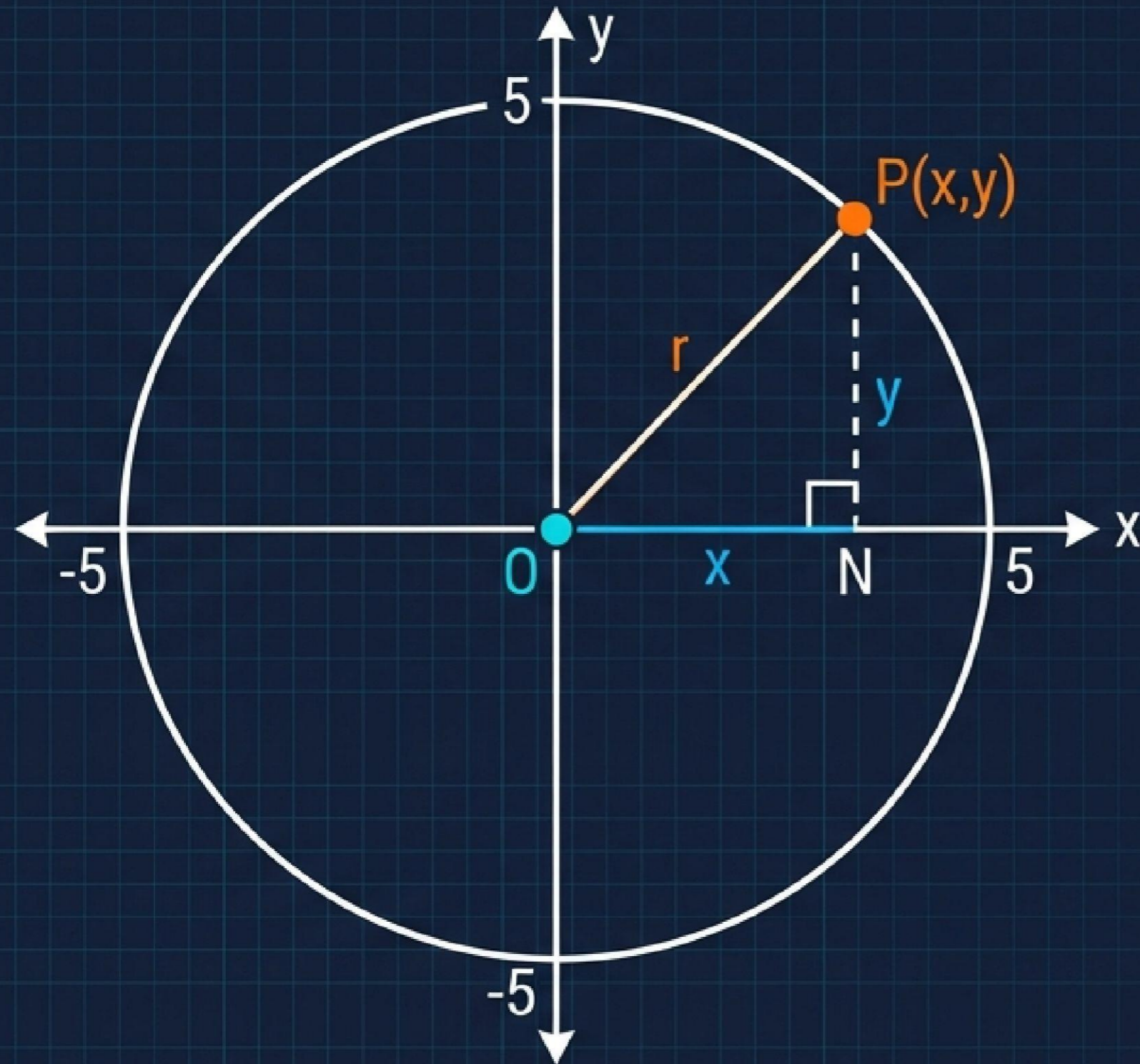
$$\text{Area of Triangle } ABC = 0$$

(Triangle has collapsed)



PRO TIP: The Area Method is often the fastest calculation.

THE GEOMETRY OF A CIRCLE



Roboto Mono

Definition: The locus of all points $P(x, y)$ at a fixed distance (radius r) from a fixed point.

Formula section:

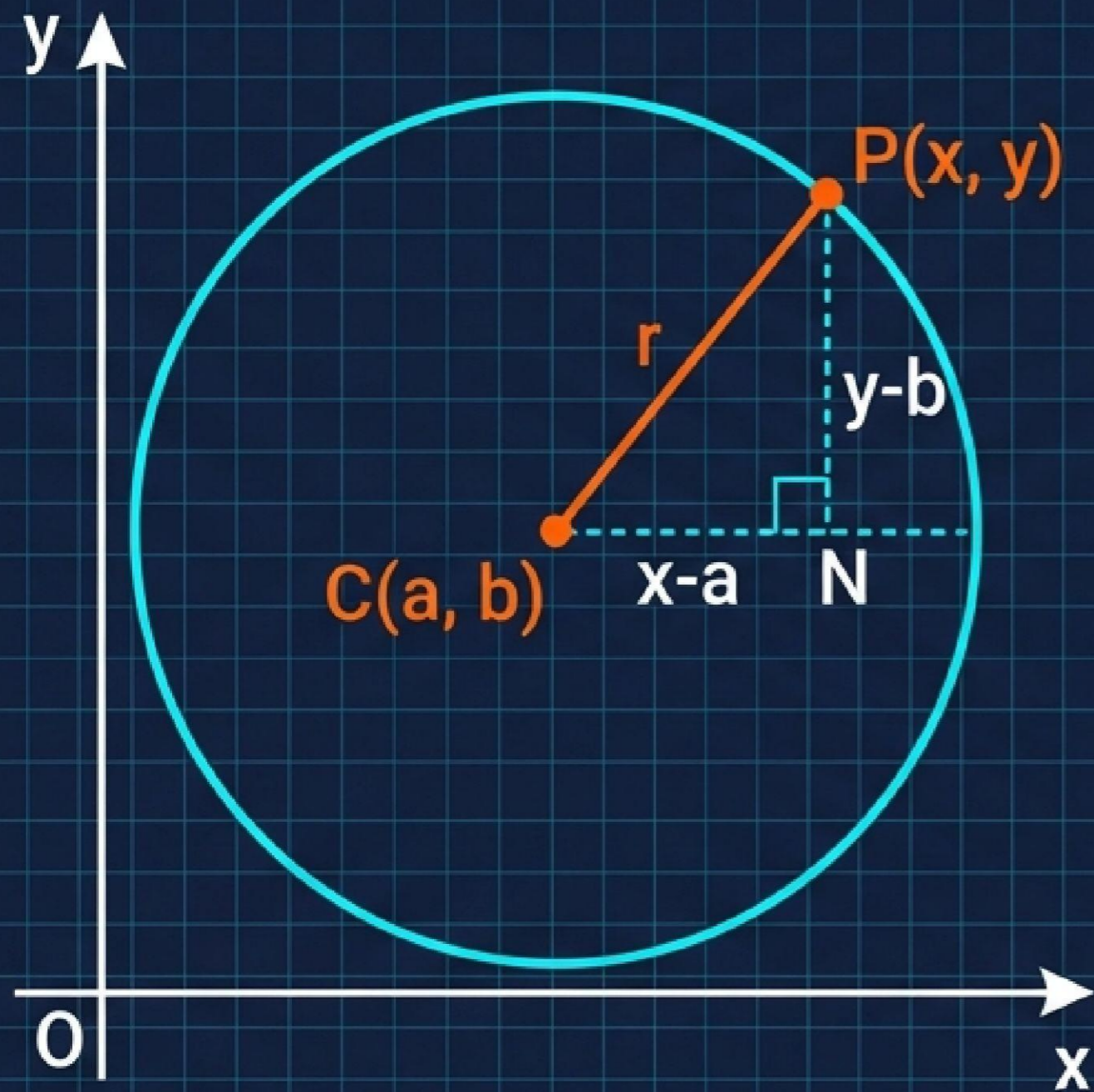
Derived from Distance Formula:
Distance from origin = r

Equation (Center at Origin):

$$x^2 + y^2 = r^2$$

Example: A circle with radius 5 is $x^2 + y^2 = 25$.

THE GENERAL CIRCLE



Shifted Center: If center is at (a, b) with radius r :

$$(x - a)^2 + (y - b)^2 = r^2$$

General Equation:

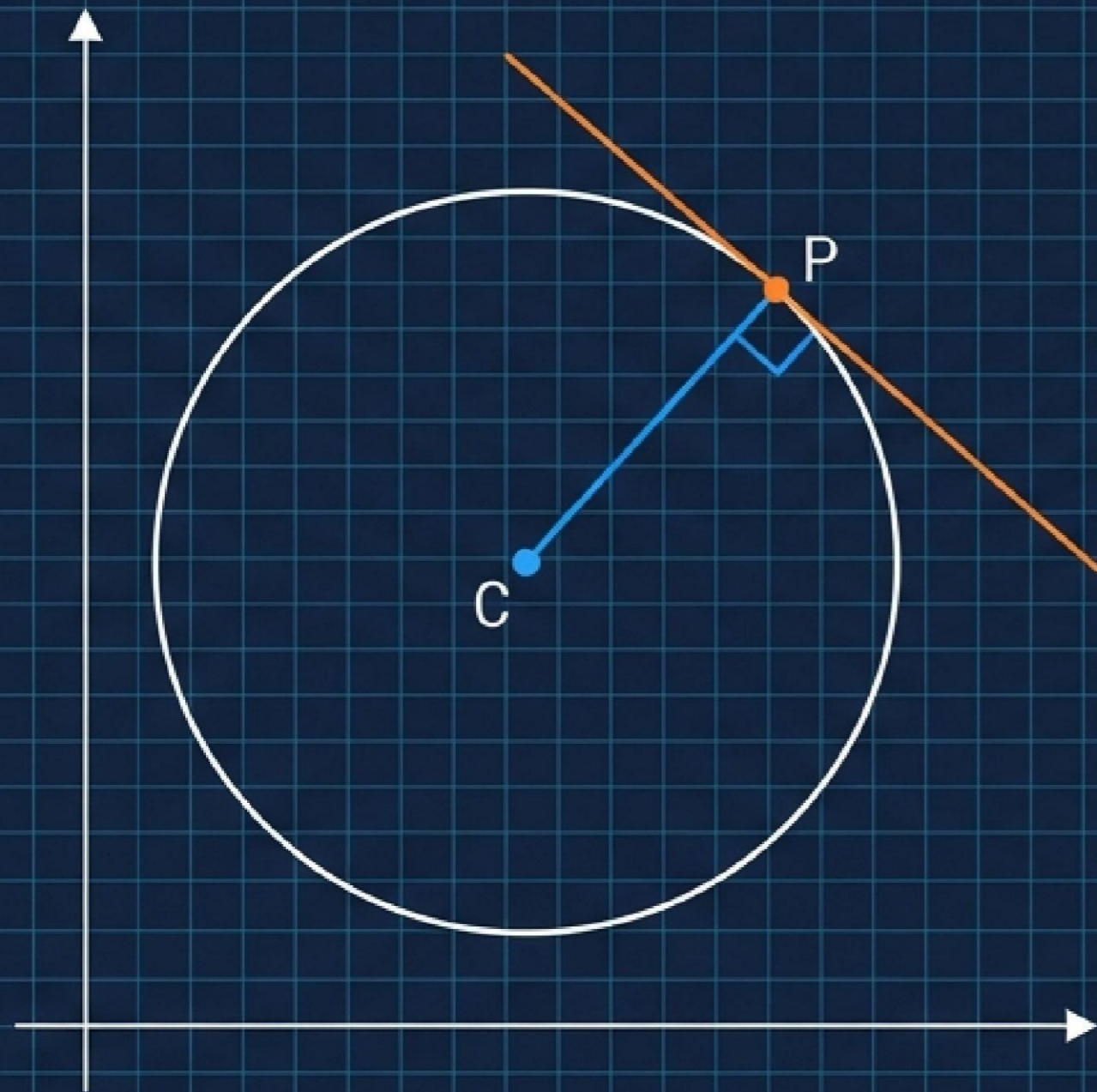
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Decoding the General Form:

- Center: $(-g, -f)$

- Radius: $r = \sqrt{(g^2 + f^2 - c)}$

TANGENTS TO A CIRCLE



Definition: A line that touches the circle at exactly one point.

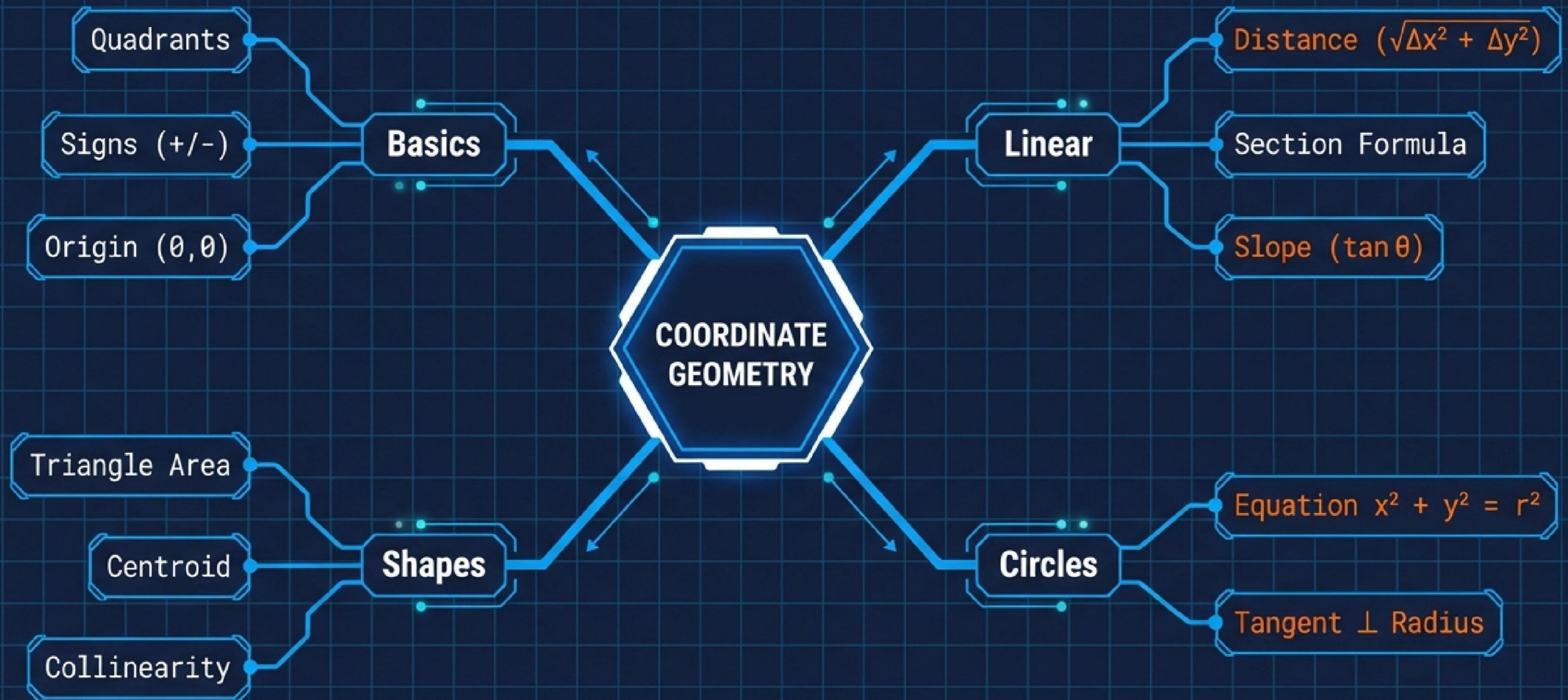
Geometric Key

The tangent is always perpendicular to the radius at the point of contact.

Calculation Steps:

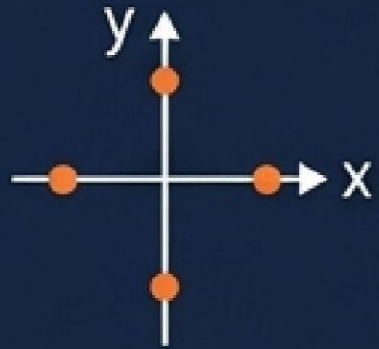
1. Find slope of the radius (m_1).
2. Slope of tangent (m_2) must satisfy $m_1 \times m_2 = -1$.
3. Use point-slope form: $y - y_1 = m_2(x - x_1)$.

SYSTEM OVERVIEW



STRATEGIC PROBLEM SOLVING

POINTS ON AXES



On x-axis: Assume $(x, 0)$.
On y-axis: Assume $(0, y)$.

FINDING RATIOS



Assume ratio is $k:1$
instead of $m:n$ to solve
for one variable.

PROVING SHAPES



Rectangle: Prove
opposite sides equal AND
diagonals equal.



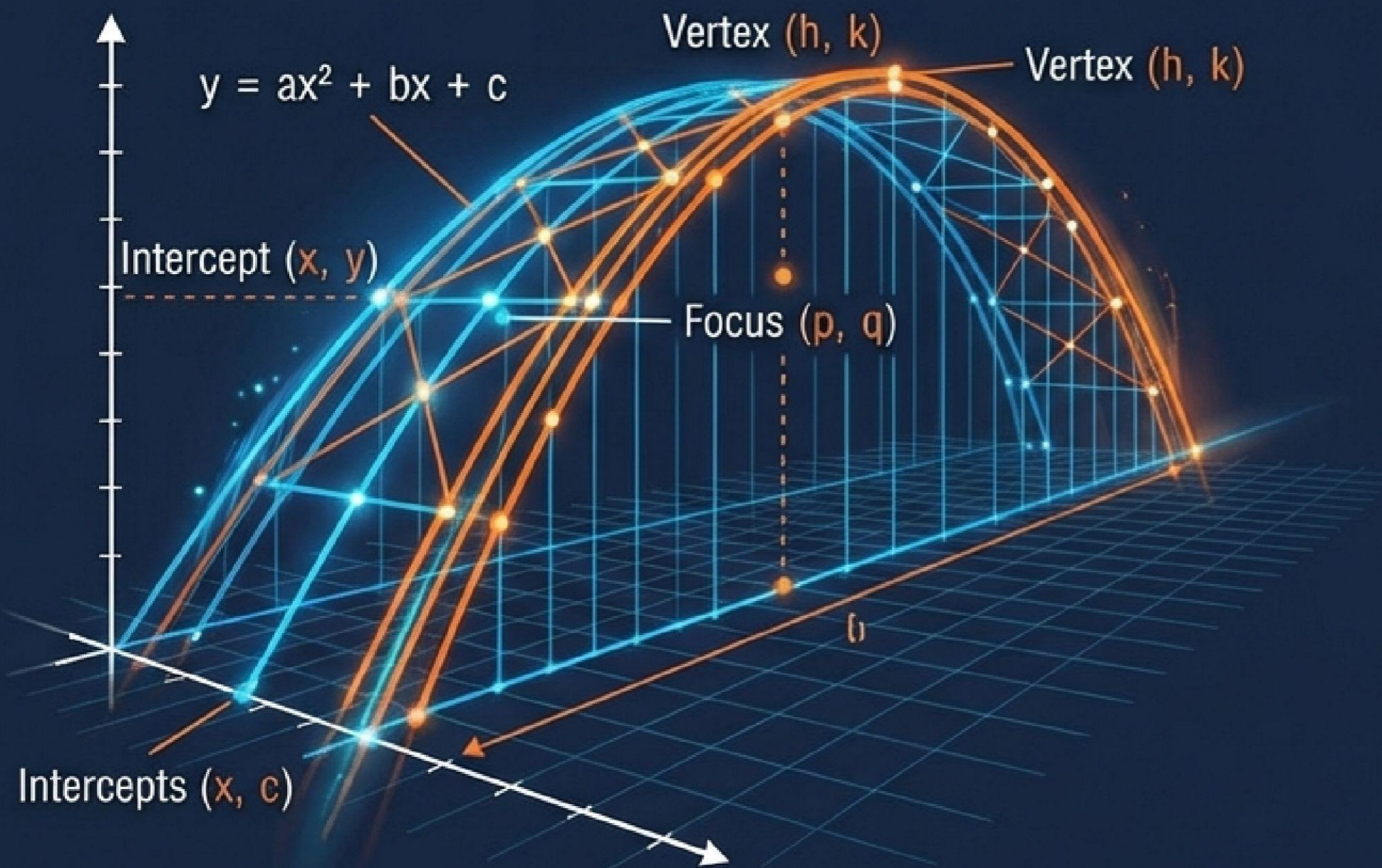
Right Triangle: Prove
 $d1^2 + d2^2 = d3^2$
(Pythagoras).

EQUIDISTANT POINTS



Set $\text{Distance}(PA) =$
 $\text{Distance}(PB)$ and square
both sides to remove
radicals.

THE FOUNDATION OF HIGHER MATH



Coordinate Geometry is the language of the physical world.

- **Physics:** Analyzing projectile motion and vectors.
- **Calculus:** Describing rates of change (**slopes**) and areas under curves.
- **Engineering:** Designing structures using precise **loci**.

By mastering the connection between numbers and space, you hold the key to modeling the universe.